Research Paper Mathematics

A Study on Divisor Cordial Labelling of Star Attached Paths and Cycles

Dr.A.NELLAI MURUGAN — Associate Professor, PG and Research Department of Mathematics, V.O.Chidambaram College,Tuticorin -628008

Miss .M.TAJ NISHA — M.S.c.,Mathematics, V.O.Chidambaram College, Tuticorin.

ABSTRACT

A divisor cordial labeling of a graph G with vertex set V is a bijection from V to {1,2,....,V(G)} such that if each edge uv is assigned the label 1 if f(u)/f(v) or f(v)/f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

In this paper ,it is proved that $P_4 @ 2K_{1,n}$, $P_6 @ 2K_{1,n}$, $C_4 \otimes S_n$, $C_5 \otimes S_n$ are divisor cordial graphs.

KEYWORDS cordial labeling, Divisor cordial graph 2010 Mathematics subject classification Number:05C78

1.Introduction:

A graph G is a finite non –empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges.Each pair e={u,v} of vertices in E is called an edge or a line of G in which e is said to join u and v.We write e=uv and say that u and v are adjacent vertices; vertex u and the edge e are incident with each other,as are v and e.If two distinct edges e_1 and e_2 are incident with a common vertex, then they are called adjacent edges. A graph with p vertices and q edges is called (p,q)-graph. By a graph ,we mean a finite simple and undirected graph.The vertex set and edge set of a graph G denoted by V(G) and E(G) respectively. For graph theoretic terminology we follow [1,2].

Definition:1.1

Let G be a graph and we define the concept of divisor cordial labeling as follows:

A divisor cordial labeling of a graph G with vertex set V is a bijection from V to {1,2,....,V(G)} such that if each edge uv is assigned the label 1 if f(u)/f(v) or f(v)/f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

Definition:1.2

$P_m @ 2K_{1,n}$ is a graph which is obtained by joining the root of the star $K_{1,n}$ to the end vertex of the path P_m .

Definition: 1.3

$C_n \otimes S_m$ is a graph which is obtained by joining the root of the star S_m to the any one vertex of the cycle C_n

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2. Main results

THEOREM 2.1 :

$P_4 @ 2K_{1,n}$ is a divisor cordial graph.

PROOF:

Let $V(P_4 @ 2K_{1,n}) = \{ (u_i / 1 \le i \le 4), (v_i, w_i : 1 \le i \le n) \}$

Let E (P_4 @ $2K_{1,n}$) =

$\{ [(u_i u_{i+1}) / 1 \le i \le 3]$

$\cup[(u_4 v_i) / 1 \le i \le n] \cup [(u_1 w_i) / 1 \le i \le n] \}$

The vertex labeling are defined by $f: V(P_4 @ 2K_{1,n}) \rightarrow \{1, 2, \ldots, 2n+4\}$

$f(u_2) = 3$

$f(u_3) = 2$

$f(w_i) = 2(i+2)\ ; \ 1 \le i \le n$

$f(v_i) = 2i+3\ ; \ 1 \le i \le n$

The induced edge labeling are,

$f(u_i u_{i+1}) = 1\ ; \ i = 1, 3$

$f(u_i u_{i+1}) = 0\ ; \ i = 2$

$f(u_1 w_i) = 1\ ; \ 1 \le i \le n$

$f(u_4 v_i) = 0\ ; \ 1 \le i \le n$

Here,

$e_f(0) = e_f(1) - 1$

Clearly, this satisfies the condition,

$| e_f(0) - e_f(1) | \le 1$

Hence, the induced edge labeling shows that P_4 @ $2K_{1,n}$ is a divisor cordial graph.

For example, P_4 @ $2K_{1,6}$ is a divisor cordial graph as shown in figure 2.2.

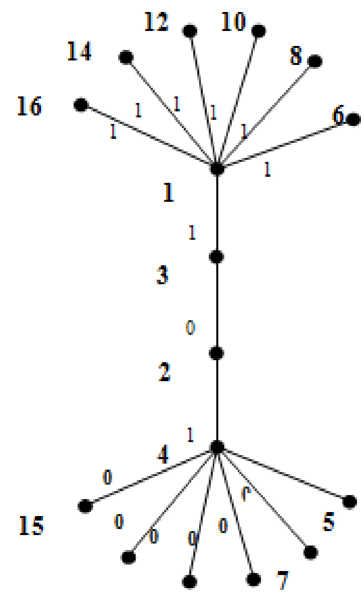

figure 2.2 : P_4 @ $2K_{1,6}$

THEOREM 2.3 :

P_6 @ $2K_{1,n}$ is a divisor cordial graph.

PROOF:

Let V (P_6 @ $2K_{1,n}$) = $\{(u_i / 1 \le i \le 6),$

$(u_{1i} , u_{2i} : 1 \le i \le n)\}$

Let E (P_6 @ $2K_{1,n}$) =

$\{ [(u_i u_{i+1}) / 1 \le i \le 5]$

$\cup[(u_1 u_{2i}) / 1 \le i \le n]$

$\cup[(u_6 u_{1i}) / 1 \le i \le n]\}$

The vertex labeling are defined by

$f : V(P_6 @ 2K_{1,n}) \rightarrow \{ 1,2,\ldots,2n+6 \}$

$f(u_i) = i \quad ; \quad i = 1, 6$

$f(u_2) = 4$

$f(u_3) = 2$

$f(u_4) = 5$

$f(u_5) = 3$

$f(u_{2i}) = 2(i+3) \; ; \; 1 \leq i \leq n$

$f(u_{1i}) = 2i+5 \quad ; \; 1 \leq i \leq n$

The induced edge labeling are,

$f(u_1u_2) = 1$

$f(u_5u_6) = 1$

$f(u_iu_{i+1}) = 0 \quad ; i = 3,4$

$f(u_iu_{2i}) = 1 \quad ; 1 \leq i \leq n$

$f(u_6u_{1i}) = 0 \quad ; 1 \leq i \leq n$

Here,

$e_f(0) = e_f(1) - 1$

Clearly, this satisfies the condition,

$| e_f(0) - e_f(1) | \leq 1$

Hence, the induced edge labeling shows that $P_6 @ 2K_{1,n}$ is a divisor cordial graph.

For example, $P_6 @ 2K_{1,6}$ is a divisor cordial graph as shown in figure 2.4.

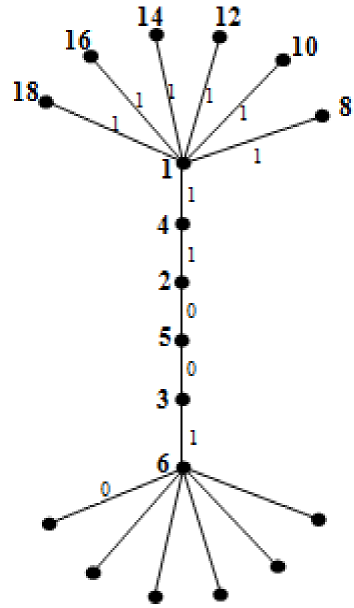

figure 2.4: $P_6 @ 2K_{1,6}$

THEOREM 2.5 :

$C_4 \otimes S_n$ is a divisor cordial graph.

PROOF:

Let $V(C_4 \otimes S_n) = \{ (u_i / 1 \leq i \leq 4),$

$v, (v_i / 1 \leq i \leq n) \}$

Let $E(C_4 \otimes S_n) = \{[(u_iu_{i+1})/1 \leq i \leq n-1]$

$\cup (u_1u_4) \cup (u_1v) \cup [(vv_i) / 1 \leq i \leq n-1] \}$

The vertex labeling are defined by

$f : V(C_4 \otimes S_n) \rightarrow \{ 1, 2, \ldots, n+4 \}$

$f(u_1) = 1$

$f(v) = 2$

$f(u_i) = n + i \; ; 2 \leq i \leq 4$

$f(v_i) = i + 2 \; ; 1 \leq i \leq n-1$

The induced edge labeling are,

$f(u_1 v) = 1$

$f(v v_i) =$

$$\begin{cases} 0 & if \quad i \equiv 1 \, mod \, 2 \\ 1 & if \quad i \equiv 0 \, mod \, 2 \end{cases} \quad 1 \leq i \leq n$$

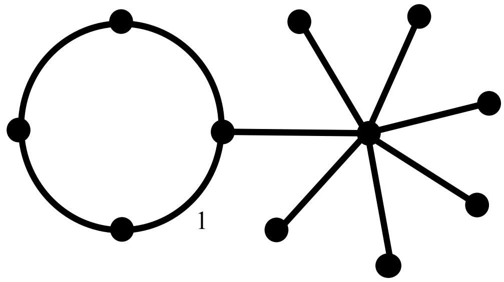

Figure 2.6 : $C_4 \otimes S_7$

When n is odd,

$e_f(0) = e_f(1) - 1$

When n is even,

$e_f(0) = e_f(1)$

Clearly, it satisfies the condition

$$|e_f(0) - e_f(1)| \leq 1$$

Hence, the induced edge labeling shows that, $C_4 \otimes S_n$ is a divisor cordial graph.

For example, $C_4 \otimes S_7$ and $C_4 \otimes S_8$ is a divisor cordial graph as shown in figure 2.6 and figure 2.7 respectively.

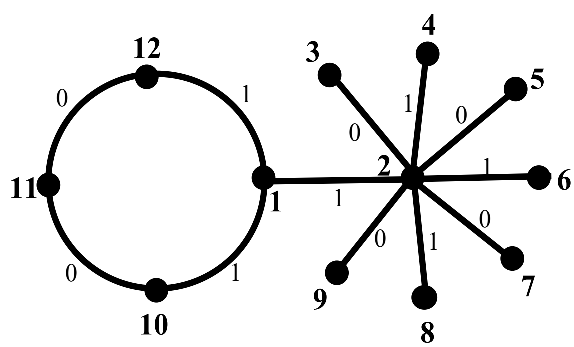

Figure 2.7 : $C_4 \otimes S_8$

THEOREM 2.8 :

$C_5 \otimes S_n$ is a divisor cordial graph.

PROOF:

Let $V(C_5 \otimes S_n) = \{ (u_i / 1 \leq i \leq 5),$

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$v, (v_i / 1 \leq i \leq n) \}$

Let $E(C_5 \otimes S_n) = \{[(u_i u_{i+1})/1 \leq i \leq n\text{-}1] \cup (u_1 u_5) \cup (u_1 v) \cup [(vv_i) / 1 \leq i \leq n\text{-}1]\}$

The vertex labeling are defined by

$f : V(C_5 \otimes S_n) \rightarrow \{1, 2, \ldots, n+5\}$

$f(u_1) = 1$

$f(v) = 2$

$f(u_i) = n + i \quad ; 2 \leq i \leq 5$

$f(v_i) = i + 2 \quad ; 1 \leq i \leq n-1$

The induced edge labeling are,

$f(u_1 v) = 1$

$f(vv_i) =$

$$\begin{cases} 0 & if \quad i \equiv 1 \bmod 2 \\ 1 & if \quad i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

When n is odd,

$e_f(0) = e_f(1)$

When n is even,

$e_f(0) - 1 = e_f(1)$

Clearly, this satisfies the condition,

$|e_f(0) - e_f(1)| \leq 1$

Hence the induced edge labeling shows that $C_5 \otimes S_n$ is a divisor cordial graph.

For example, $C_5 \otimes S_7$ and $C_5 \otimes S_8$ is a divisor cordial graph as shown in figure 2.9 and figure 2.10 respectively.

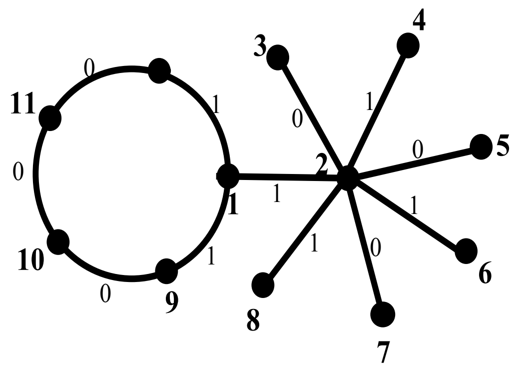

Figure 2.9: $C_5 \otimes S_7$

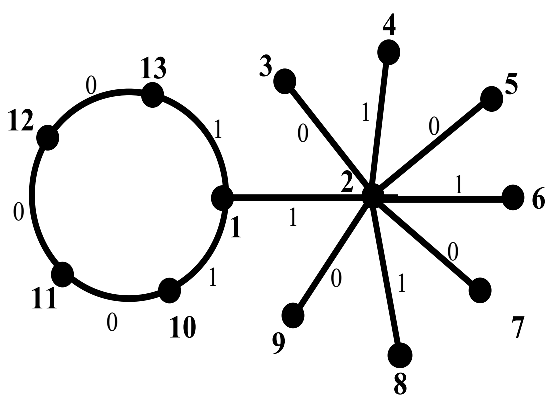

Figure 2.10 : $C_5 \otimes S_8$

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