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"Fuzzification of Bisection Method"

Mathematics

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G. C. Hazarika	Professor, Department of Mathematics, Dibrugarh University
Ms Toralima Bora	Lecturer, Centre for Computer Studies, Dibrugarh University, Dibrugarh.

ABSTRACT

Fuzzy logic deals with reasoning that is approximate rather than fixed and exact. The term "fuzzy logic" was introduced in conjunction with the proposal of fuzzy set theory by lotfi A. Zadeh in the year 1965. Fuzzy logic has been applied to many fields. In mathematics and statistics, a fuzzy variable is a value which could lie in a probable range defined by quantitative limits or parameters and which can be usefully described with imprecise categories. Researchers in the past investigated a number of methods of numerical analysis with the help of Fuzzy theory. In this paper Bisection method is fuzzified using fuzzy interval.

In this paper an attempt has been made to fuzzify the Bisection Method to find the root of an algebraic equation. Results have been observed by fuzzifying the intervals and its midpoint with iteration method. Proceeding in this way fuzzy membership functions can be found at next successive points using Fuzzy Bisection Method.

Introduction

Fuzzy logic deals with reasoning that is approximate rather than fixed and exact. The term "fuzzy logic" was introduced in conjunction with the proposal of fuzzy set theory by lotfi A. Zadeh in the year 1965.Fuzzy logic has been applied to many fields. In mathematics and statistics, a fuzzy variable is a value which could lie in a probable range defined by quantitative limits or parameters and which can be usefully described with imprecise categories. Researchers in the past investigated a number of methods of numerical analysis with the help of Fuzzy theory. In this paper Bisetion method is fuzzified using fuzzy interval.

Preliminaries of Bisection Method

The bisection method in mathematics is a root-finding method that repeatedly bisects an interval and then selects a subinterval in which a root must lie for further processing. It is a very simple and robust method, but it is also relatively slow.

Let us consider an equation $f(x) = 0$.

Let $f(x)$ be continuous and it can be algebraic or transcendental. Let the function $f(x)$ changes sign over an interval $x = a$ and $x = b$. Then there is a root of $f(x) = 0$ lying between a and b.

As a first approximation, the root of $f(x) = 0$ is $x_0 = \frac{a+b}{2}$.

Suppose $f(a)$ and $f(x_0)$ are of opposite signs then the root lies between a and x_0 and if $f(x_0)$ and $f(b)$ are of opposite sign then the root lies between x_0 and b.

If $f(a) < 0$ and $f(b) > 0$ then the first approximation be $x_0 = \frac{a+b}{2}$. Suppose $f(x_0) < 0$, then the root lies between x_0 and b. Then the second approximation is $x_1 = \frac{x_0 + b}{2}$. Suppose $f(x_1)$ is positive . Then the root lies between x_0 and x_1 and the third approximation is $x_2 = \frac{x_0 + x_1}{2}$ and so on.

This method is simple but slowly convergent.

Fuzzification of Bisection Method

Let us consider an equation $F(X) = 0$.

Let $F(X)$ be continuous and it can be algebraic or transcendental. Let the function $F(X)$ changes sign over an interval $X = A$ and $X = B$. Let A=$[A_1, A_2, A_3]$ and B=$[B_1 , B_2, B_3]$.Then there is a root of $F(X)$=0 lying between A and B.

Now f.m.f. of A and B are respectively

$$\mu_A(X) = \begin{cases} \frac{X - A_1}{A_2 - A_1} & where\ A_1 \le X \le A_2 \\ \frac{X - A_3}{A_2 - A_3} & where\ A_2 \le X \le A_3 \\ 0 & otherwise \end{cases}$$

$$\mu_B(X) = \begin{cases} \frac{X - B_1}{B_2 - B_1} & where\ B_1 \le X \le B_2 \\ \frac{X - B_3}{B_2 - B_3} & where\ B_2 \le X \le B_3 \\ 0 & otherwise \end{cases}$$

with respect to α-cuts as

$$[A]^\alpha = [A_1 + \alpha(A_2 - A_1), A_3 + \alpha(A_2 - A_3),]$$

$$[B]^\alpha = [B_1 + \alpha(B_2 - B_1), B_3 + \alpha(B_2 - B_3),]$$

As a first approximation, the root of $F(X) = 0$ is

$$X_0 = \frac{A+B}{2} = \frac{[A_1, A_2, A_3] + [B_1 , B_2, B_3]}{2}.$$

Let us consider $X_0 = [X_0', X_0'', X_0''']$.

The f.mf of X_0 is

$$\mu_{X_0}(X) = \begin{cases} \frac{X - X_0'}{X_0'' - X_0'} & where\ X_0' \le X \le X_0'' \\ \frac{X - X_0'''}{X_0'' - X_0'''} & where\ X_0'' \le X \le X_0''' \\ 0 & otherwise \end{cases}$$

with respect to α-cut

$$[X_0]^\alpha = [X_0' + \alpha(X_0'' - X_0'), X_0''' + \alpha(X_0'' - X_0'''),]$$

Suppose $F(A)$ and $F(X_0)$ are of opposite signs then the root lies between A and X_0 and if $F(X_0)$ and $F(B)$ are of opposite sign then the root lies between X_0 and B.

If $F(A) < 0$ and $F(B) > 0$ then the first approximation be

$$X_0 = \frac{A+B}{2} = \frac{[A_1, A_2, A_3] + [B_1 , B_2, B_3])}{2}.$$

Suppose $F(X_0) < 0$, then the root lies between X_0 and B

Then the second approximation is

$X_1 = \frac{X_0+B}{2} = \frac{[X_0',X_0'',X_0'']+[B_1,B_2,B_3]}{2}$.

Let us consider $X_1 = [X_1', X_1'', X_1''']$

The f.m.f. of X_1 is

$$\mu_{X_1}(X) = \begin{cases} \dfrac{X - X_1'}{X_1'' - X_1'} & where\ X_1' \le X \le X_1'' \\ \dfrac{X - X_1'''}{X_1'' - X_1'''} & where\ X_1'' \le X \le X_1''' \\ 0 & otherwise \end{cases}$$

with respect to α-cut

$$[X_1]^\alpha = [X_1' + \alpha(X_1'' - X_1'), X_1''' + \alpha(X_1'' - X_1'''),]$$

Suppose $F(X_1)$ is positive. Then the root lies between $X_0$and X_1

and the third approximation is

$X_2 = \frac{X_0+X_1}{2} = \frac{[X_0',X_0'',X_0'']+[X_1',X_1'',X_1'']}{2}$where

$X_2 = [X_2', X_2'', X_2'']$

The f.m.f. of X_2 is

$$\mu_{X_2}(X) = \begin{cases} \dfrac{X - X_2'}{X_2'' - X_2'} & where\ X_2' \le X \le X_2'' \\ \dfrac{X - X_2'''}{X_2'' - X_2'''} & where\ X_2'' \le X \le X_2''' \\ 0 & otherwise \end{cases}$$

with respect to α-cut

$$[X_2]^\alpha = [X_2' + \alpha(X_2'' - X_2'), X_2''' + \alpha(X_2'' - X_2'''),]$$

and so on.

This method is simple but slowly convergent.

A Numerical Example

Let us consider the algebraic equation

$$F(X) = X^3 - 4X - 9$$

Let A=[1.99,2,2.01] and B=[2.99,3,3.01]

the function $F(X)$ changes sign over an interval $X = A$ and $X = B$.

Here $F(A) < 0$ and $F(B) > 0$.

Therefore there is a root of $F(X)$ =0 lying between A and B.

Then

$X_0 = \frac{1}{2}\{A + B\} = \frac{1}{2}\{[1.99,2,2.01] + [2.99,3,3.01]\}$
$= [2.49,2.5,2.51]$

Now f.m.f. of X_0 is

$$\mu_{X_0}(X) = \begin{cases} \dfrac{X - 2.49}{2.5 - 2.49} & where\ 2.49 \le X \le 2.5 \\ \dfrac{X - 2.51}{2.5 - 2.51} & where\ 2.5 \le X \le 2.51 \\ 0 & otherwise \end{cases}$$

with respect to α-cut

$$[X_0]^\alpha = [2.49 + \alpha(2.5 - 2.49), 2.51 + \alpha(2.51 - 2.5$$

$F(X_0) = F([2.49,2.5,2.51]) < 0$

So the root lies between X_0 and B

$X_1 = \frac{1}{2}\{X_0 + B\} = \frac{1}{2}\{[2.49,2.5,2.51] + [2.99,3,3.01]\} = [2.74,2.75,2.76]$

the f.m.f. of X_1

$$\mu_{X_1}(X) = \begin{cases} \dfrac{X - 2.74}{2.75 - 2.74} & where\ 2.74 \le X \le 2.75 \\ \dfrac{X - 2.76}{2.75 - 2.76} & where\ 2.75 \le X \le 2.76 \\ 0 & otherwise \end{cases}$$

with respect to α-cut

$$[X_1]^\alpha = [2.74 + \alpha(2.75 - 2.74), 2.76 + \alpha(2.75 - 2.76)]$$

$F(X_1) = F([2.74,2.75,2.76]) > 0$

So the root lies between X_0 and X_1.

$$X_2 = \frac{1}{2}\{X_0 + X_1\} = \frac{1}{2}\{[2.49,2.5,2.51] + [2.74,2.75,2.76]\} = [2.615,2.625,2.635]$$

The f.m.f. of X_2 is

$$\mu_{X_2} = \begin{cases} \dfrac{X - 2.615}{2.625 - 2.615} & where\ 2.615 \le X \le 2.625 \\ \dfrac{X - 2.635}{2.625 - 2.635} & where\ 2.625 \le X \le 2.635 \\ 0 & otherwise \end{cases}$$

with respect to α-cut

$$[X_2]^\alpha = [2.615 + \alpha(2.625 - 2.615), 2.635 + \alpha(2.625 - 2.635)]$$

$F(X_2) = F([2.615,2.625,2.635]) < 0$

Therefore the root lies between X_1 and X_2

$$X_3 = \frac{1}{2}\{X_1 + X_2\} = \frac{1}{2}\{[2.74,2.75,2.76] + [2.615,2.625,2.635]\} = [2.6775,2.6875,2.6975]$$

The f.m.f. of X_3 is

$$\mu_{X_3}(X) = \begin{cases} \dfrac{X - 2.6775}{2.6875 - 2.6775} & where\ 2.6775 \le X \le 2.6875 \\ \dfrac{X - 2.6975}{2.6875 - 2.6975} & where\ 2.6875 \le X \le 2.6975 \\ 0 & otherwise \end{cases}$$

with respect to α-cut

$$[X_3]^\alpha = [2.6775 + \alpha(2.6875 - 2.6775), 2.6975 + \alpha(2.6875 - 2.6975)]$$

Now

$F(X_3) = F([2.6775,2.6875,2.6975])<0$

So the root lies between X_1 and X_3

$X_4 = \frac{1}{2}\{X_1 + X_3\}$

$= \frac{1}{2}\{[2.74,2.75,2.76] + [2.6775,2.6875,2.6975]\}$
$= [2.70875,2.71875,2.72875]$

The f.m.f. of X_4 is

$$\mu_{X_4}(X) = \begin{cases} \frac{X - 2.708755}{2.71875 - 2.70875} & where\ 2.70875 \le X \le 2.71875 \\ \frac{X - 2.72875}{2.71875 - 2.72875} & where\ 2.71875 \le X \le 2.72875 \\ 0 & otherwise \end{cases}$$

with respect to α-cut

$$[X_4]^\alpha = [2.70875 + \alpha(2.71875 - 2.70875), 2.72875 + \alpha(2.71875 - 2.72875)]$$

and so on.

Conclusion

In this paper a attempt has been made to fuzzify the Bisection Method to find the root of an algebraic equation. Results have been observed using fuzzy interval and the midpoint under consideration. Proceeding in this way fuzzy membership functions can be found at next successive points using Fuzzy Bisection Method.

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