Volume : 3 | Issue : 6 | June 2014 • ISSN No 2277 - 8179

Research Paper

New Technique to Detect Redundant Constraints in Large Scale Linear Programming Problems

Mathematics

KEYWORDS : *linear programming, redundant constraints, binding constraints, feasible region, restrictive constraint, robust reduction*

P.Sumathi	Research Scholar, Department of Mathematics, Easwari Engineering College, Ramapuram, Chennai - 600 089
A.Gangadharan	Professor & Research Supervisor, Department of Mathematics, Easwari Engineering College, Ramapuram, Chennai - 600 089

ABSTRACT

Linear programming finds many uses in the business and industry, where a decision maker may want to utilize limited available resources in the best possible manner. The limited resources may include material, money, manpower, space and time. Linear programming provides various methods of solving such problems. In this paper a new approach is proposed to identify the redundant constraints in linear programming problems and compared with the existing three methods and analyzing the computational efforts - work (efforts changed to work) by solving various sizes of linear programming problems.

1. Introduction

Many researchers [1 - 13] have proposed different methods to identify the redundancies in linear programming problems. Ioslovich [6] proposed new methods to identify redundant constraints. These methods consume more number of computational efforts - work (efforts changed to work) and time. To reduce the time and computational effort this paper suggests a new approach to select a restrictive constraint which is presented in the secction 2. Section 3 describes the earlier methods with one numerical example. Comparison results of four methods are presented in section 4. Section 5 concludes the paper.

Redundant constraint

A redundant constraint is a constraint that can be removed from a system of linear constraints without changing the feasible region.

Consider the following system of m non-negative linear inequality constraints and n variables (m≥n).

$AX \leq b, X \geq 0$ ------- (1)

where $A \in R^{m \times n}$, $b \in R^m$, $X \in R^n$ and $0 \in R^n$.

Let $A_iX \leq b_i$ be the i^{th} constraint of the system (1) and let $S = \{X \in R^n / A_iX \leq b_i, X \geq 0\}$ be the feasible region associated with system (1).

Let $S_k = \{X \in R^n / A_iX \leq b_i, X \geq 0, i \neq k\}$ be the feasible region associated with the system of equations $A_iX \leq b_i$, $i = 1, 2, m$, $i \neq k$. The k^{th} constraint $A_kX \leq b_k$ ($1 \leq k \leq m$) is redundant for the system (1) if and only if $S = S_k$.

Redundant constraints can be classified as weakly and strongly redundant constraints.

Weakly redundant constraint

The constraint $A_iX \leq b_i$ is weakly redundant if it is redundant and $A_iX = b_i$, for some $X \in S$.

Strongly redundant constraint

The constraint $A_iX \leq b_i$ is strongly redundant if it is redundant and $A_iX < b_i$, for all $X \in S$.

Binding constraint

Binding constraint is the one which passes through the optimal solution point. It is also called a relevant constraint.

Non-binding constraint

Non-binding constraint is the one which does not pass through the optimal solution point. But it can determine the boundary of the feasible region.

2. Proposed Method

In this section, a new approach is suggested to select the most restrictive constraint. The steps of the proposed method are as follows.

Let us consider the following problem

$$\text{Max } Z = \sum_{j=1}^{n} c_j x_j$$

Subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$,

i = 1,2,3,...,m

$0 \leq x_j \leq u_j$, j = 1,2,3,...,n

Step 1:
Divide the left hand side value by each of the resource constraints along the respective

co- ordinate axis. Where $= \overline{a_{ij}} = b_i / a_{ij}, (b_i > 0, \forall_i)$

Step 2:
Compute $S_i = \sum_{j=1}^{n} |\overline{a_{ij}}|$ for each i∈I, I= {1,2,3,...,m}

Step 3:
Select a most restrictive constraint corresponding to

l. Where $l = \arg\min_i (S_i)$,

1 ≤ | ≤ m

(1 less than or equal to l(small letter el (l)) less than or equal to m.

Step 4:
Identified the constraints $A_lX \leq b_k$, is redundant if $\alpha_k^l < b_k$ Where α_k^l is the optimal value of LP_k^l, where LP_k^l is

LP_k^l: Maximize $\alpha_k^l = A_lX$

Subject to $A_lX \leq b_l$

$0 \leq X \leq U$

2.1. Numerical Illustration

Example 1:

Maximize $z = 60x_1 + 70x_2 + 15x_3$

Subject to $3x_1 + 6x_2 + 4x_3 \leq 3400$ ----- 1

$5x_1 + 6x_2 + 7x_3 \leq 3600$ ----- 2

$3x_2 + 4x_2 + 5x_3 \leq 2600$ ----- 3

$x_1 + x_2 + x_3 \leq 38000$ ----- 4

$x_1, x_2, x_3 \geq 0$

Volume : 3 | Issue : 6 | June 2014 • ISSN No 2277 - 8179

Solution:
Here

Here $C = (60\ 70\ 15)$

$$A = \begin{pmatrix} 3 & 6 & 4 \\ 5 & 6 & 7 \\ 3 & 4 & 5 \\ 1 & 1 & 1 \end{pmatrix}$$

$$b^T = (3400\ 3600\ 2600\ 3800)U^T$$
$$= (720\ 560\ 514)$$

$S_1 = 2550$
$S_2 = 1834$
$S_3 = 2036$
$S_4 = 11400$

Maximize $\alpha_k^l = A_l X$
Subject to $A_l X \le b_l$, where $l = \arg\min_i(S_i)$,
$l = 2$
$0 \le X \le U$

(i) $\alpha_1^2 = (3\ 6\ 4)$

$$(5\ 6\ 7)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \le 3600$$

$\alpha_1^2 = 3504$, constraint 1 is not redundant constraint.

(ii) $\alpha_3^2 = (3\ 4\ 5)$

$$(5\ 6\ 7)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \le 3600$$

$\alpha_3^2 = 2571$, constraint 3 is redundant constraint.

(iii) $\alpha_4^2 = (1\ 1\ 1)$

$$(5\ 6\ 7)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \le 3600$$

$\alpha_4^2 = 720$, constraint 4 is redundant constraint.

Therefore Constraint 3, 4 are redundant constraints.

3. Ioslovich methods

Method 1:

Maximize $\alpha_i^f = \alpha_i' x$
Subject to $c^T x \le Z_l$
$0 \le x \le u$
If $\alpha_i^f < b_i$ then the i^{th} constraint is redundant.

Step 1:
To find z_i values, solve the problem
Maximize $z_i = c^T x$
Subject to $\alpha_i' \le b_i$,

Step 2:
Find $y_{u'A}$ and $y_{u'l}$.

Step 3:
Find Max $z_u = c^T x$,
Subject to $y_u' Ax \le y_u' b$
$0 \le x \le u$

Step 4:
$Z_l = \min(z_{il}, z_u)$
where $z_{il} = \min_i z_i$.

Example:
Consider the example of section 2.1.

Z_i values are 57666, 43200, 50900, 90110

Where $y_{1u} = 11.67$, $y_{2u} = 11.67$, $y_{3u} = 17.50$, $y_{4u} = 0$. By step 3

$z_u = 50588.86$. Where $z_{il} = 43200$. Then $Z_l = 43200$

$y_{u'A} = (145.86\ \ 210.04\ \ 215.87)$ and $y_{u'l} = (127190)$

Since values are 5098, 6640, 4598, 1105

Constraint 4 only identified as redundant constraint by this method for the above example.

Method 2:
Step 1: where

Where Z_i is the optimal value of LP_i. Where LP_i is

LP_i: Max $Z_i = C^T X$

Subject to $A_i X \le b_i$,

$0 \le X \le U$

Step 2:

Example:
Maximize $\alpha_i^k = a_i' x$
Subject to $\alpha_k' x \le b_k$
$0 \le x \le u$
If $\alpha_i^k < b_i$, then the i^{th} constraint is redundant.

Consider the example 1of section 2.1.

where, $k = \arg\min Z_i$, z = z1, z2, z3, … zm, k = 2.

Z_i values are 57666, 43200, 50900, 90110

Here values are 3504, 2571, 720.

Therefore constraints 3, 4 are redundant.

Method 3:
Solve the problem

Maximize $\alpha_i^\alpha = \alpha_i' x$
Subject to $y_u' Ax \le y_u' b$
$0 \le x \le u$
If $\alpha_i^\alpha < b_i$ then the i^{th} constraint is redundant.

Example:
Consider the example of section 2.1

Where $y_{1u} = 11.67$, $y_{2u} = 11.67$, $y_{3u} = 17.50$, $y_{4u} = 0$.

$y_{u'A} = (145.86\ \ 210.04\ \ 215.87)$ and $y_{u'l} = (127190)$

Here α_i^Ω values are 3556.78, 4318.93, 2903.87, 825.56

Constraint 4 is redundant constraint.

4. Comparison results of four methods

In this paper efficiency of the four methods has been discussed. The above three methods take more computational efforts - work and time compared with the proposed method. Table 4.1 shows the comparison of identification of redundant constraints for small scale problems. From this table (4.1) method 2 and the proposed method detects same number of redundant constraints. Table 4.2 shows the number of operations (multiplications/division) and time taken for both proposed and Ioslovich's 2nd method for small scale problems in micro seconds. Table 4.3, 4.4 shows the same comparison for large scale and netlib problems respectively. The comparison results are clearly shown below.

TABLE 4.1: COMPARISON OF FOUR METHODS (Small Scale Problems)

S. NO.	Size of the problem		Number of Redundant Constraints Identified by(Redundant constraint number)			
	No. of Constraints	No. of Variables	Method 1	Method 2	Method 3	Proposed Method
1	3	2	1(3)	1(3)	1(3)	1(3)
2	3	2	-	-	-	-
3	3	2	1(3)	1(3)	1(3)	1(3)
4	4	3	1(4)	2(3,4)	1(4)	2(3,4)
5	4	3	2(1,4)	2(1,4)	1(4)	2(1,4)
6	3	3	1(3)	1(3)	1(3)	1(3)
7	3	3	1(2)	1(2)	1(2)	1(2)
8	4	5	1(4)	1(4)	1(4)	1(4)
9	5	4	-	2(2,4)	1(4)	2(2,4)
10	7	10	1(2)	5(2,3,4,6,7)	5(2,3,4,6,7)	5(2,3,4,6,7)

TABLE 4.2: COMPARISON OF TWO METHODS (Small Scale problems)

S. NO.	Size of the problem		Ioslovich		Proposed	
	No. of Constraints	No. of Variables	No. of Mult/ Div.	Time (Micro Seconds)	No. of Mult/ Div.	Time (micro Seconds)
1	3	2	747	201	326	179
2	3	2	815	286	326	187
3	3	2	747	203	326	184
4	3	3	2034	285	648	185
5	3	3	1895	290	786	200
6	4	3	2682	306	972	231
7	4	3	2681	295	1248	235
8	4	5	107860	643	3795	336
9	5	4	5082	460	2724	349
10	7	10	221226	8516	94146	3797

TABLE 4.3: COMPARISON OF TWO METHODS (Large Scale Problems)

S. NO.	Problem Name	Size of the problem		Ioslovich		Proposed	
		No. of Constraints	No. of Variables	No. of Mult/ Div.	Time (Milli Seconds)	No. of Mult/ Div.	Time (milli Seconds)
1	scpe1	50	500	6690755821	79819	4714	619
2	scpe2	50	500	6374861569	73176	5013	728
3	scpe3	50	500	6457362052	77843	5017	731
4	scp410	200	1000	35672031823	1290345	5588	1231
5	scpcyc06	240	192	1125643192	475412	960	456
6	scpclr10	511	210	2215762319	625342	13130	932
7	scpcyc07	672	448	24113547041	978823	2688	548
8	scpclr11	1023	330	63298164027	23674983	41424	2896

TABLE 4.4: COMPARISON OF TWO METHODS (Netlib Problems)

S. NO.	Problem Name	Size of the problem		Ioslovich		Proposed	
		No. of Constraints	No. of Variables	No. of Mult/ Div.	Time (Milli Seconds)	No. of Mult/ Div.	Time (micro Seconds)
1	stocfor1	62	62	1037722762	12386	3844	368
2	scsd1	77	77	1300501684	35726	5929	474
3	share1b	102	102	96317594	24507	10404	319
4	bandm	180	180	763358312	467894	32400	773
5	scrs8	181	181	398996542	518239	32761	862
6	qfrdpnc	322	322	1503029648	761302	103684	1021
7	czprob	475	475	1202387492	894763	225625	1285
8	perold	500	500	1172688049	5401371	250000	1765
9	scfxm3	728	728	1660987246	7646783	529984	2894

5. CONCLUSION

In this paper, three methods of robust reduction have been compared with proposed method. Each method has its own role in viewing computational effort - work and time factor. From table 4.1, we observe that the method 2 and proposed method gives the same result. But the proposed method is less time consuming as compared to method 2. The proposed method requires small as less number of computational steps. Therefore it is easy to identify the redundant constraints even in a large scale linear programming problems.

REFERENCE

[1] Brown, G.W., & Koopmans, T.C. (1951). Computa tional suggestions for maximizing a linear function | subject to linear inequalities, in T.C.Koopmans, | editor, Activity analysis of production and aloction | John Wiley, New York , 377-380. | | [2] Corley, H.W., Jay Rosenberger., Wei-Chang Yeh | & Sung, T.K. (2006). The cosine simplex | algorithm. The international Journal of advanced | manufacturing Technology, 27,1047-1050. | | | [3] Danny C Myers. (1992). A dual simplex Imple | mentation of a constraint selection algorithm for | linear programming, The Journal of the operational | research society, 43, 177-180. | | [4] Danny C Myers., & Wei Shih. (1998). A constraint selection technique for a class of linear programs, | Operation Research letters, 7, 191-195. | | [5] Karmarker, N. (1984). A new polynomial time | algorithm for linear programming, Combinatorica, | 4, 373-395. | [6] Ilya Ioslovich (2001).Robust reduction of a class of large scale linear programs, SIAM journal of | optimization, 12, 262–282. | | [7] Murty, K.G., & Fathi, Y. (1984). A feasible directi on metod for linear programming, Operations | Research Letters, 3, 121-127. | | [8] Paulraj S., Chellappan, C., & Natesan, T.R.(2006). A heuristic approach for identification of redundant constraints in linear programming methods, | International Journal of computer mathematics, 83, | 675-683. | | [9] Paulraj, S., & Sumathi, P. (2010). A comparative | study of redundant constraints identification meth ods in linear programming problems, Mathematical Problems in Engineering, 2010, Article ID | 723402, 16 pages. | | [10] Stojkovic, N.V., & Stanimirovic, P.S. (2001).Two direct methods in linear programming,European | Journal of operational research. 131, 417-439. | | [11] Sumathi, P., & Paulraj, S. (2013). Identification | of redundant constraints in large scale linear | pogramming problems with minimal | computa-tionaleffort, Applied Mathematical | Sciences, 7, 3963-3974. | | [12] Wei – Chang., Yen., &Corley, H.W.(2009). | A simple direct cosine simplex algorithm, Applied | mathematics and computation, 214, 178-186. | | [13] Yamamota, Y. (2010), A new method for | solving a linear programming problem,49th IEEE | conference on decision and control, December | 15-17, 3891-3895. |