Stagnation Point Flow of MHD Nanofluid Over A Stretching Sheet with Effect of Heat Source/Sink, Momentum, Thermal and Solutal Slip



Mathematics

**KEYWORDS :** MHD Nano-fluid, Stagnation point, Slip parameters, Heat Source/ Sink.

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ABSTRACT In pres	ent paper we studied the effect of heat source/sink on flow and heat transfer of MHD nanofluid due		

to stretching sheet. The governing PDE's of the flow heat transfer and mass transfer are converted into ODE's using suitable similarity transformations, with its boundary conditions. The ODE's of the flow, heat and mass transfer are solved numerically using Fourth order Runge-Kutta method with efficient shooting technique. The effect of governing parameters on flow, heat and mass transfer are studied using the plots The various numerical tables which are calculated and tabulated. A comparison of our present results with a previous published works has been done and we found that an excellent agreement is there with the earlier results and of ours.

# 1. INTRODUCTION

Mustafa et al [1] studied the flow heat and mass transfer of nanofluid due to stretching sheet. Here the governing equations of the flow heat and mass transfer are solved by Homotopy Analysis Method and studied effects of all governing parameters on flow, heat and mass transfer. Mukinde and Aziz [2] to study the effect of a convective boundary condition on boundary layer flow, heat and mass transfer and nanoparticle fraction over a stretching surface in a nanofluid. The governing boundary layer equations have been transformed to a two-point boundary value and are solved numerically.

Anuar Ishak et .al[3] the steady two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature. In this paper the governing system of partial differential equations are transferred into ordinary differential equations, which are solved numerically using a finite-difference scheme known as the Keller-box method. The effects of the governing parameters on the flow field and heat transfer characteristics are obtained.

Norfiftah Bachok et.al[4] studied the two-dimensional stagnation point flow of a water based nanofluid over an exponentially stretching/shrinking sheet. Z.Abbas et.al[5] the steady mixed convection boundary layer flow of an incompressible Maxwell fluid near the two-dimensional stagnation-point flow over a vertical stretching surface and it is assumed that the stretching velocity and the surface temperature very linearly with the distance from the stagnation point. The homotopy Analysis Method (HAM) and the influence of the various interesting parameters on the flow and heat transfer is analyzed. Hassani et. al[6] studied the boundary layer flow heat and mass transfer of nanofluid due to past a stretching sheet. Here the governing equations of the flow heat and mass transfer are solved by Homotopy Analysis Method and studied effects of all governing parameters on boundary layer flow, heat and mass transfer. Kuznetsov and Nield [7] to studied the natural convection boundary layer flow, heat and mass transfer of nanofluid due to past a vertical plate. Here the governing equations of the flow heat and mass transfer are solved by analytical method and studied effects of all governing parameters on natural convection boundary layer flow, heat and mass transfer. Noghrehabadi et. al [8] to analyze the slip effects on the boundary layer flow and heat transfer over a stretching surface of nanoparticle fractions. Here the governing equations of slip effects on the boundary layer flow and heat transfer are solved by numerically. The effects of slip boundary condition in the presence of dynamic effects of nano particle have been investigated. Kandasamy et. al[9] to study the boundary layer flow, heat transfer and nanoparticle volume fraction over a stretching surface in a nanofluid for various parameters

using scalling group of transformation. Bhattacharyya and Vajravelu[10] invesigated the boundary layer stagnation point flow and heat transfer over an exponentially shrinking sheet. Here an exponential form of similarity transformation, the governing mathematical equations for the flow and heat transfer are transformed into self-similar coupled, nano-linear ordinary differential equations. Rohni et. al[11] the flow and heat transfer over an unsteady shrinking surface with wall mass suction in a nanofluid by using an appropriate similarity transformation, similarity equations are obtained and the shooting method is used to solve these equations for different values of the wall mass suction, unsteadiness nanofluid parameters. Bhattacharyya [12] studied the heat transfer in unsteady boundary layer stagnation point flow over a shrinking/stretching. The governing equations are transformed into self-similar ordinary differential equations by adopting similarity transformations and then the converted equations are solved numerically by shooting method.

Yacob et. al [13] studied the boundary layer stagnation point flow of a micropolar fluid towards a horizontally linearly stretching/shrinking sheet. Here a mathematical model is devolved to study the heat transfer characteristics occurring during the melting process due to a stretching/shrinking sheet. The transformed non-linear ordinary differential equations governing the flow are solved numerically by the Runge-Kutta -Fehlberg method with shooting technique. Layek et al[14] the study of two-dimensional stagnation point flow of an incompressible viscous fluid towards a porous stretching surface embedded in a porous medium subject to suction/blowing with internal heat generation or absorption. The motion of this study is to explore the influence of suction/blowing on the control of flow separation as well as heat transfer and also to investigate the effects of heat source or sink parameter on hear transfer. The momentum and thermal boundary layer equations are solved numerically using shooting method.

Turkyilmazoglu and Pop[15] the flow and heat transfer of a Jeffrey fluid near the stagnation point on a stretching/shrinking sheet with a parallel external flow. The main concern is to analytically investigate the structure of the solutions which might be unique or multiple. Heat transfer analysis is also carried out for a boundary heating process taking into consideration both a uniform wall temperature and a linearly increasing wall temperature. Hayat et. al [16] studied the two-dimensional stagnation point flow of an incompressible fluid over a stretching sheet by taking into a account radiation effects using the Rosseland approximation to model the radiative heat transfer. Under suitable similarity variables, the partial differential equations are transformed into a system of non-linear ordinary differential equations which is solved analytically by the homotopy Analytic Method (HAM). Ibrahim et. al [17] studied the effect of magnetic field on stagnation point flow and heat transfer due to nanofluid towards a stretching sheet. The transport equations employed in the analysis include the effect of parameters. The similarity transformation is used to convert the governing nonlinear boundary layer equations to coupled higher order nonlinear ordinary differential equation. These equations were numerically solved using Runge-Kutta fourth order method with shooting technique. Mahapatra et.al[18] studied the Analytical solution of magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching sheet. Here the governing equations of the flow heat and mass transfer are solved by Homotopy Analysis Method and studied effects of all governing parameters on flow, heat and mass transfer.

#### 2. MATHEMATICAL FORMULATION:

The governing equations of flow heat and mass transfer of considered fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
(1)  

$$\frac{u}{2\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho f} \frac{\partial p}{\partial x} + v (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + \frac{\sigma B_0^2}{\rho} (U_\infty - u) f$$

$$\frac{u}{2\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho f} \frac{\partial p}{\partial y} + v (\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) - \frac{\sigma B_0^2}{\rho} f v$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau D_B \left(\left(\frac{\partial \phi}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y}\right)\frac{D_T}{T_\infty}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right]\right) + \frac{q^{m}}{\rho c_p}$$

$$(4)$$

$$u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} = D_B(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}) + \frac{D_T}{T_{\infty}}(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2})$$

Where u and v are velocity components along x and yaxis  $\upsilon$  is the kinematics viscosity  $\rho_f$  is the density of the base fluid, s electrical conductivity  $U_{*}$ ,  $B_{0}$ ,  $\rho p$ ,  $(\rho c)_{f}$ ,  $D_{s}$  and  $D_{r}$  are the free stream velocity, , magnetic field, the density of the nanoparticle, heat capacity of a base fluid, the Brownian diffusion and thermophoretic diffusion coefficient respectively, and

$$\alpha = \frac{k}{(\rho c)_f}, \tau = \frac{(\rho c)_p}{(\rho c)_f}, \upsilon = \frac{\mu}{\rho f}.$$
  
Where  $\underline{q}^{m} = \frac{kb}{\upsilon} [A^*(T_w - T_w)f' + B^*(T - T_w)],$ 

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V= (u, v),  $\rho_{nf}$ ,  $\mu_{nf}$ ,  $k_{nf}$ , and  $\beta_{nf}$  are the density, the thermal conductivity, and the volumetric volume expansion coefficient of the nanofluid, respectively, which are defined as Thermophysical Properties of nanofluids are given by

$$(\rho c_{p})_{nf} = (1-\phi)(\rho c_{p})_{f} + \phi(\rho c_{p})_{p}$$

$$\frac{k_{nf}}{k_{f}} = \frac{k_{p} + 2k_{f} - 2\phi(k_{f} - k_{p})}{k_{p} + 2k_{f} + 2\phi(k_{f} - k_{p})}$$

$$(6)$$

$$\rho_{nf} = (1-\phi)\rho_{f} + \phi\rho_{p}$$

$$\mu_{nf} = \frac{\mu_{f}}{(1-\phi)^{2.5}}$$

$$(\rho\beta)_{nf} = (1-\phi)_{f} + \phi(\rho\beta)_{p}$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_{p})_{nf}}$$

$$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_p 
 \frac{k_{nf}}{k_f} = \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + 2\phi(k_f - k_p)}$$
(7) (8)

 $\phi$  is the Solid volume fraction, T is the temperature inside the boundary layer,  $(\rho c)_p$  effective heat capacity of a nanofluid, and  $\mu$  is is the acceleration due to gravity.  $\mu_f$  is the dynamic iscosity of the base fluid,  $\beta_f$  and  $\beta_p$  are the thermal expansion coefficients of the base fluid and the nanoparticle, respectively, and  $\beta_p$  are the densities of the nanoparticle, the suffixes f, p, and nf denote base fluid, nanoparticle, and nanofluid conditions, respectively, and  $(\rho c_p)_{nf}$  is the heat capacitance of the nanofluid, Where  $\vec{k}_f$  and kp are the thermal conductivities of the base fluid and nanoparticle, respectively.

The boundary conditions are:

$$u - u_{w}(x) = L \frac{\partial u}{\partial y}, \quad v = 0,$$
  

$$T - T_{w}(x) = k_{1} \frac{\partial T}{\partial y}, \quad C - C_{w}(x) = k_{2} \frac{\partial C}{\partial y} \quad \text{at } y = 0,$$
  

$$u \to 0, \quad T \to T_{\infty}, \quad C \to C_{\infty} \quad as \quad y \to \infty,$$
  
Where  $u_{w}(x) = \alpha x, \quad T_{w}(x) = T_{\infty} + b(\frac{x}{l}) \text{ and}$ 

$$C_w(x) = C_\infty + c(\frac{x}{l})$$

Using the transformation

$$u = \alpha x f'(\eta), v = -\sqrt{\alpha v} f(\eta),$$

$$\frac{T - T_{\infty}}{T_{w} - T_{\infty}} = \theta(\eta), \quad h(\eta) = \frac{\phi - \phi_{\infty}}{\phi_{w} - \phi_{\infty}}$$
  
Where  $\eta = \sqrt{\frac{\alpha}{\nu}}y$ ,

Using an order magnitude analysis of the y-direction momentum equation (normal to the sheet) using the usual boundary layer approximation:

$$u \gg v; \quad \frac{\partial u}{\partial y} \gg \frac{\partial v}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial p}{\partial y} = 0 \quad (10)$$

After boundary layer approximation, the governing equations are reduced to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{11}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_{\infty}\frac{\partial U_{\infty}}{\partial x} + v(\frac{\partial^2 u}{\partial y^2}) + \frac{\sigma B_0^2}{\rho}(U_{\infty} - u)f$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial y^2}\right) + \tau \left\{D_B\left(\frac{\partial h}{\partial y}\frac{\partial T}{\partial y}\right)\frac{D_T}{T_{\infty}}\left[\left(\frac{\partial T}{\partial y}\right)^2\right]\right\} + \frac{q^{m}}{\rho c_p}$$

(13)

$$u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} = D_B(\frac{\partial^2 h}{\partial y^2}) + \frac{D_T}{T_{\infty}}(\frac{\partial^2 T}{\partial y^2}) \qquad (14)$$

The boundary conditions are also reduced

$$u-u_w(x)=L\frac{\partial u}{\partial y}, \quad v=0,$$

$$T - T_w(x) = k_1 \frac{\partial T}{\partial y}, \quad C - C_w(x) = k_2 \frac{\partial C}{\partial y} \quad \text{at}$$

$$u \to 0, \quad T \to T_{\infty}, \quad C \to C_{\infty} \quad as \quad y \to \infty,$$
  
Where  $u_w(x) = \alpha x, \quad T_w(x) = T_{\infty} + b(\frac{x}{l})$ 

 $C_{w}(x) = C_{\infty} + c(\frac{x}{l}).$ and

We now introduce the following dimensionless quantities; i.e similarity transformations used to reduce given partial differential equations to ordinary differential equations.

$$\eta = \sqrt{\frac{\alpha}{\nu}} y, \qquad \psi = -\sqrt{\alpha \nu x} f(\eta),$$
$$\theta(\eta) = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \quad h(\eta) = \frac{\varphi - \varphi_{\infty}}{\phi_{w} - \phi_{\infty}}$$
(16)

The equations of continuity is satisfied if we choose a stream function  $\mathcal{W}$  (xy) such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{17}$$

Using the similarity transformation quantities, the governing equations (11). (12), (13) and (14) are transformed to the ordinary differential equation as follows:

$$f''' + ff'' - f'^{2} + M(A - f') + A^{2} = 0$$
(18)

$$\theta'' + \Pr f \theta' + \Pr Nth' \theta' + \Pr Nb\theta'^2 + A^* f + B^* \theta = 0$$
(19)

$$h'' + Lefh' + \frac{Nt}{Nb}\theta'' = 0 \tag{20}$$

With the boundary conditions

$$f(0) = 0, \quad f'(0) = 1 + \beta f''(0),$$
  

$$\theta(0) = 1 + \gamma \theta'(0), \quad h(0) = 1 + \delta \phi'(0),$$
  

$$f'(\infty) = A, \qquad \theta(\infty) = 0, \text{ as } \eta \to \infty$$

Where the six governing parameters are defined as:

$$Pr = \frac{\vartheta}{\alpha}, \qquad A = \frac{b}{a}, \quad M = \frac{\sigma B_0^2}{\rho_a \alpha}, Nb = \frac{(\rho c)_p D_B (\varphi_w - \varphi_\infty)}{(\rho c)_f \vartheta}$$

$$Nb = \frac{(\rho c)_p D_T (T_w - T_{00})}{(\rho c)_f \vartheta T_{00}}, \quad Le = \frac{\vartheta}{D_B}$$
$$Nb = \frac{(\rho c)_p D_T (T_w - T_{00})}{(\rho c)_f \vartheta T_{00}}, \quad Le = \frac{\vartheta}{D_B}$$

Where f',  $\theta$  and h are the dimensionless velocity, temperature and particle concentration with respect to η is the similarity variables, the prime denotes differentiation with respect to n. Pr, A, M, Nb, Nt, Le denotes a Prandtl number, and a velocity Volume : 3 | Issue : 6 | June 2014 • ISSN No 2277 - 8179

rario, a magnetic parameter, a Brownian motion parameters, a thermophoresis parameter, and a Lewis number, respectively.

The important physical quantities of interest in this problem are the skin friction coefficient cf, local Nusselt number  $N_{u}$  and the local Sherwood number Sh, are defined as:

$$c_{f} = \frac{\tau_{w}}{\rho(u_{w})^{2}}, Nu_{x} = \frac{xq_{w}}{k(T_{w} - T_{\infty})},$$
$$Sh_{x} = \frac{xh_{m}}{D_{B}(\phi_{w} - \phi_{\infty})}$$
(23)

Where the skin friction  $\, {\cal T}_{_W} \,$  , wall heat flux qw and wall mass flux  $q_w$  are given by

$$\tau_{w} = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_{w} = -k \left(\frac{\partial T}{\partial y}\right)_{y=0},$$

$$h_{m} = -D_{B} \left(\frac{\partial \phi}{\partial y}\right)_{y=0}, \quad (24) \quad \text{By}$$
using the above  $d\mathbf{\hat{u}}$  ations, we get

$$c_f \sqrt{\operatorname{Re}_x} = f''(0), \quad \frac{Nu_x}{\sqrt{\operatorname{Re}_x}} = -\theta'(0),$$

$$\frac{Sh_x}{\sqrt{\operatorname{Re}_x}} = -h'(0) \quad (25)$$

where  $\operatorname{Re}_x Nu_x$ ,  $Sh_x$  are local Reynolds number, local Nusselt number and local Sherwood number respectively.

#### **3. NUMERICAL SOLUTION:**

In this study, an efficient Runge-kutta fourth order method along with shooting technique has been used to analyze the flow model of coupled ordinary differential equations Eq.(18)-(20) for different values of governing parameters viz. prandtl number Pr, velocity ratio parameter A, a Brownian motion parameter Nt and a Lewis number Le. The coupled ordinary differential equations (18)-(20) are third order in f and second order in both  $\theta$  and h respectively which have been reduced to a system of seven simultaneous equations for seven unknown. In order to solve numerically this system of equations using Range-kutta method, we require seven initial conditions but two initial conditions in f one initial condition in each of  $\theta$  and h are known. However, the values of f',  $\theta$  and h are known at  $\eta \rightarrow \infty$ . Thus, these three end conditions are utilized to produce two unknown initial conditions at  $\eta = 0$  by using shooting technique.

The Equations (18)-(20) can be expressed as

$$f^{""} = -ff^{"} + f^{'2} - M(A - f^{'}) - A^{2}$$
(26)

$$\theta^{*} = -\Pr f \theta^{'} - \Pr Nth^{'}\theta^{'} - \Pr Nb\theta^{'2} - A^{*}f \theta - B^{*}\theta$$
(27)

$$h^{"} = -Lefh^{'} - \frac{Nt}{Nb}\theta^{"}$$
 (28)

Now defining new variables by the equation

$$f_1 = f, f_2 = f', f_3 = f'', f_4 = \theta, f_5 = \theta',$$
  
 $f_6 = h, f_7 = h',$  (29)

The three coupled higher order differential equations and the boundary conditions may be transformed to seven equivalent

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first order differential equations and boundary conditions respectively as given below: Using Eq.(29) we can write the initial value problem as follows:

$$\begin{array}{cccc} f_{1'} & & f_{2} \\ f_{2} & & f_{3}^{2} \\ f_{3}' & & -f_{1}f_{3} + f_{2}^{2} - M(A - f_{2}) - A^{2} \\ f_{4}' & = & f_{5} \\ f_{5}' & & \end{array}$$

$$-\Pr_{(30)} f\theta' - \Pr_{Nth'\theta'} - \Pr_{Nb\theta'^2} - A^* f \theta - B^* \theta$$

$$f_6' f_7' - Le + \frac{Nt}{Nb} (\Pr_{f_1} f_5 + \Pr_{Nb} h_5 f_7 + \Pr_{Nt} h_5^2)$$
(31)

A prime denote the differentiation with respect to  $\eta$  and the boundary conditions are Here prime denotes the differentiation with respect to and the initial conditions in Eq.(21) become as follows:

Here we need to solve a sequence of initial value problems as above, by taking  $\alpha = \alpha_n$ ,  $\beta = \beta_n$  and  $\gamma = \gamma_n$ , so that the end boundary values thus obtained numerically match up to desired degree of accuracy with the boundary values at  $\infty$  given in the problem. In what follows,  $f_i(\infty, \alpha, \beta, \gamma)$  is the solution at infinity to be obtained by the classical Runge-kutta method for unknown slopes. Let us assume the initial value problem satisfies necessary conditions for existence and uniqueness of solutions, the problem now reduces to that of finding  $\alpha$ ,  $\beta$  and  $\gamma$  such that:

$$F(p,q,r) = f'(\infty, p,q,r) - f'(\infty) = f_2(\infty, p,q,r) - f_2(\infty) = 0$$
  

$$\Theta(p,q,r) = \theta(\infty, p,q,r) - \theta(\infty) = f_4(\infty, p,q,r) - f_4(\infty) = 0$$
  

$$H(p,q,r) = h(\infty, p,p,r) - h(\infty) = f_6(\infty, p,q,r) - f_6(\infty) = 0$$
  
(32)

These are three non-linear equations in p, q and r which are to be solved by Newton-Rephson method. This method for finding roots of non-linear equations, with  $p_0, q_0$  and  $r_0$  as initial values, which yields the following iterative scheme:

$$\begin{bmatrix} p_{n+1} \\ q_{n+1} \\ r_{n+1} \end{bmatrix} = \begin{bmatrix} p_n \\ q_n \\ r_n \end{bmatrix} - \begin{bmatrix} \frac{\partial F}{\partial p} \frac{\partial F}{\partial q} \frac{\partial F}{\partial r} \\ \frac{\partial \theta}{\partial p} \frac{\partial \theta}{\partial q} \frac{\partial \theta}{\partial r} \\ \frac{\partial H}{\partial p} \frac{\partial H}{\partial q} \frac{\partial H}{\partial r} \end{bmatrix}_{p_n, q_n, r_n} \begin{bmatrix} F \\ \Theta \\ H \end{bmatrix}_{p_n, q_n, r_n}$$
(33)

0,1,2,3.....

To implement the scheme (33) we require finding the nine partial derivatives in the to be inverted matrix. These can be

obtained by differentiating the initial value problem, given in Eq.(30)-(31) with respect to p, q and r. By differentiating Eq.(30)-(31) with respect to p, q and r we get three more initial value problem known as variational equations. On solving some initial value problems, it is possible to go ahead with iterative scheme (33).

The accuracy chosen for obtaining p, q and r Newton-Raphson method was  $10^{-7}$ . convergence of Newton-Raphson iterative scheme was ensured due to the scientific choice of missing initial values there by circumventing the usual problem of slow convergence or divergence or overflow encountered in shooting method procedures. Here we also note that, the governing ODEs of stretching sheet problems are not stiff and hence we do not need procedures

### 4. RESULTS AND DISCUSSION:

Fig.2 illustrates the influence of velocity ratio parameter 'A' on velocity graph. When the free stream velocity exceeds the velocity of the stretching sheet, the flow velocity increases the boundary layer thickness decrease with increase in 'A'. Moreover, when the free stream velocity less than stretching velocity, the flow flied velocity decreases and boundary layer thickness also decreases. When A>1, the flow has a boundary layer structure and boundary thickness decreases as a values of 'A' increases. On the other hand, when A< 1, the flow has an inverted boundary layer structure, for this case also, as the values A decrease the boundary layer thickness decreases. Fig.3 shows that the presence of transverse magnetic field sets in Lorentz force, which results in retarding force on the velocity field. Therefore, as the values of M increases does the retarding force and hence the velocity decreases. Fig.4 depicts the variation of temperature graph with respect to Prandtl number PR. On observed the temperature decreases when the value of Prandtl number PR increase. This is due to the fact that a higher Prandtl number fluid has relatively low thermal conductivity, which reduces conduction and thereby the thermal; boundary layer thickness; and as a result, temperature decreases. The influence of prandtl

non Newtonian fluids is similar to what we observed in nanofluid. Therefore, these properties are also inherited by nanofluids. Figs.5 & 6 show the influence of the change of Brownian motion parameter NB and thermophoresis parameter NT on temperature profile respectively. It is noticed that as thermophoresis parameter increases the thermal boundary layer thickness increases and the temperature gradient at the surface decrease (in absolute value) as both NB and NT increase. As it is noticed from Fig.7 as Lewis number increases the concentration graph decreases. Moreover, the concentration boundary layer thickness decreases as Lewis number increase. This is probably due to the fact mass transfer rate increases as Lewis number increases. It also reveals that the concentration gradient at surface of the plate increases. Fig.8 depicts the effect of heat source/sink parameters which is temperature dependent. On observing the graph temperature enhance as increase in the parametric values of A\* heat generates as A\* increases. Fig.9 depicts the effect of B\* an heat transfer, it is also showing the same result as in case of A\*. Figs 10, 11 & 12 show the effect of momentum slip  $\beta$  on velocity profile, thermal slip  $\gamma$  on temperature profile and solutal slip  $\delta$  on mass transfer profile respectively, temperature profile and concentration profile decreases with increase in parametric values. But opposite result observed in velocity profile. Table 1 is tabulated for numerical values of f" (0) at M=0,  $\beta = \gamma = \delta = 0$ , our present results are compared with Ibrahim et. al [17], Mahapatra [18] and Hayat [16]. Our results are in good agreement with above said earlier works. f''(0) increase as increase in A.

**Table 2** tabulated for the numerical values of Nusselt number for various values of A and PR a Nusselt number increases on increase in values of PR and A these results are also in good agreement with earlier results Ibrahim et. al [17], Mahapatra [18] and Hayat [16]. Table 1: comparison of values of f" (0) with previous result when  $\beta = \gamma = \delta = 0$ .  $1\beta = \gamma = \delta = 0.1$ 

Table 2: Comparison of local Nusselt number -  $\theta'(0)$  at Nt=0,

Nb $\rightarrow$ 0, for different values of with previously published data.

The table 1 and 2 show the comparison study of our results with earlier work there is a good agreement between our and pervious results with table 1 earlier work.

### 5. CONCLUSION:

On investigation of nano fluid flow due to stagnation point flow, we came to the following conclusions, which are

- 1. Velocity increases with an increases in A when A > 1.
- 2. The thickness of velocity boundary layer decreases with an increase in magnetic field parameter M.
- 3. Thermal boundary layer thickness decreases with an increase in both velocity ratio parameter A and prandtl number Pr.
- 4. The thickness of thermal boundary layer increases with an increase in both Nt= Nb parameter.
- 5. The magnitude of the skin friction coefficient f"(0) increases with M when A≠1and it is zero when A=1
- An increase in velocity ratio parameter A increases both the local Nusselt number and Local Sherwood number.
- An increase in magnetic parameter M increases both the local Nusselt number - θ'(0) and local Sherwood number - h'(0).
- When the value of velocity ratio parameter A=1, the skin friction coefficient, local Nusselt number and local Sherwood number all are constant.
- 9. The wall temperature gradient increases with an increase in Lewis number Le and prandtl number Pr.
- 10. Increasing value of A\* and B\* enhances the boundary layer thickness and enhances the temperature.





# Fig-1 Schematic of the two-dimensional stretching sheet problem

Table.1

Pr	А	Present Result	Ibrahim et al [17]	Mahap et al [	atra 18]	Hayat et al [16]
1 1.5	0.1 0.2 0.5 0.1 0.2 0.5	0.6021 0.6244 0.6924 0.7768 0.7970 0.8648	0.6022 0.6245 0.6924 0.7768 0.7971 0.8648	0.603 0.625 0.692 0.777 0.797 0.863	0.60 0.62 0.69 0.77 0.79 0.86	2156 4467 2460 76802 7122 4771











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## Acknowledgement:

Authors are thankful to the reviewers for their useful comments and suggestions. One of the author Dr. Jagadish Tawade wishes to thank Bharat Ratna Prof. C.N.R.Rao, Hon'ble Chairman, Dr. S. Anant Raj Consultant, Prof. Roddam Narasimha, Hon'ble member VGST, Department of IT, BT S & T, GoK, India, for supporting this work under Seed Money to Young Scientists for Research (F.No.VGST/P-3/ SMYSR/ GRD-286 / 2013-14).

η

Fig-10

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