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Optimization of Zero point Bottleneck Transportation problem with fuzzy parameters

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ABSTRACT This paper consider an optional solution bottleneck transportation problem based on blocking zero point method. This work analyses the blocking method for finding the solution under fuzzy environment. Our model has two criteria, namely cost and time is minimized. Since, usually a transportation pattern optimizing two objectives simultaneously does not exist, we define as algorithm under blocking zero point method. At the end, the numerical example is presented to illustrate the proposed method.

KEYWORDS : Bottleneck transportation problem; fuzzy parameter; blocking method; cost and time.

1. Introduction

The classical transportation problem is defined by minimization of variable transportation costs while meeting a set of demands from a set of available supplies. It is also known as the cost and time minimizing transportation problem. One well studied variant of the classical transportation problems known as the bottleneck transportation problems. The time minimizing or bottleneck transportation problem is a special case of a transportation problem in which a time is associated with each shipping route. Rather than minimizing cost, the objective is to minimize the maximum time to transport all supply to the destination. Earlier researchers [3],[4] developed various algorithm for solving time minimizing transportation problems. The transportation time is relevant in a variety of real transportation problems too. Bottle neck-cost transportation problem (BCTP) is a kind of bicriteria transportation problem. The bicriteria transportation problem is a particular case of multi-objective transportation problem which had been proposed and also solved by Aneja and Nair [2] and until, now many researchers, [6],[7] also, have great interest in this problem and some method used their special techniques in finding the solutions for two objectives functions approaching to the ideal solution.

2. Formulation of bottleneck transportation problems (BTP):

Consider the following BTP

$$\text{Maximize } Z = [\text{maximize}\, T_{ij}/X_{ij} > 0]$$

$$\text{Subject to } \sum_{j=1}^{m} X_{ij} = a_{ij},\ i = 1,2,\ldots\ldots\ldots m \quad (1)$$

$$\sum_{i=1}^{n} X_{ij} = b_{ij},\ j = 1,2,\ldots\ldots n (2)$$

$$X_{ij} \geq 0 \text{ for all } i \text{ and } j (3)$$

Where 'm' refers the number of supply points, 'n' is the number of demand points. X_{ji} is the number of units shipped from supply point 'i' to the demand point 'j', Tij is the time of transporting goods from supply point 'i' to demand point 'j', a_{ij} is the supply at the supply point i and b_j is the demand at the demand point 'j'.

In a BTP, time matrix $[T_{ij}]$ is gives where T_{ij} is the term of transporting goods from the origin 'i' to the destination j.

For any given feasible solution $X = \{X_{ij},\ i = 1,2,\ldots..m, j = 1,2,\ldots..n\}$ of the problem the time transportation is the maximum of T_{ij}'s among the cells in which there are positive allocations. This time of the transportation remains independent of the amount of commodity sent as long as $X_{ij} > 0$

2.1 Algorithms: blocking zero point method:

In this section, we present on algorithms for Blocking Zero-point approach. The blocking approach proceeds as follows

Step: 1 Find the maximum of the minimum of each row and each column of the given transportation table.(say) T

Step: 2 construct a reduced transportation table from the given table by blocking all cells having time more than T.

Step: 3 Check if each column demand is less than to the sum of the supplies in the reduced transportation problem obtained from the step 2..Also, check if each row supply is less than to sum of the column demands in the reduced transportation problem obtained from the step 2..If so, go step 6. (Such reduced transportation table is called the active transportation table)
If not, go to step 4.

Step: 4 find a time which is immediately next to the time T (say) V.

Step: 5 Construct a reduced transportation table from the given transportation table by blocking all cells having time more than V and then, go to the step 3.

Step: 6 Do allocation according to the following rules:
(I) Allot the maximum possible to a cell which is only one cell in the row/column. Then, modify the active transportation table and then, repeat the process till it is possible or all allocations are completed.

(ii) If (i) is not possible select a row/column having minimum number of unblocked cell and allocate maximum possible to a cell which helps to reduce the large supply and/or large demand of the cell.

Step:7 The allotment yields a solution to the given bottleneck transportation problem.

Now, we prove the solution to a BTP obtained by the blocking method is an optimal solution to the BTP.

2.2 Theorem: A solution obtained by the blocking approach to the BTP is an optimal.

Proof: Let T_o be the time transportation of a feasible solution of the given problem.

Let Tα be the time transportation of the feasible solution to the given BTP by the blocking method. It means that all transportation can be made in the time Tα. In the active table maximum time isTα.

By step 3-5, Tα is the minimum time to transport all items from the origin to destinations. If $T_o \geq T_\alpha$, the solution obtained by the blocking method is an optimal,

If $T_o \leq T_\alpha$, it means that all transporting work can be made in the time of T_o and $T_o \leq T_\alpha$ therefore, T_o is a time for transportation which is less than T_α, which is a contradiction.

Thus the time for transportation obtained using the blocking method to the BTP is an optimal.
Hence the theorem.

2.3 Numerical example;
Let us consider the following transportation table
Allot the maximum possible to a cell which is only one cell in the row/column. Then, modify the active transportation table and then, repeat the process till it is possible or all allocations are completed.

(ii)
If (i) is not possible select a row/column having minimum number of unblocked cell and allocate maximum possible to a cell which helps to reduce the large supply and/or large demand of the cell.

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Hence the theorem.

2.3 Numerical example;
Let us consider the following transportation table

Table-1

								Supply
	12	13	34	7	8	29	19	15
	7	18	36	40	38	6	10	7
	11	20	30	21	21	29	31	45
	27	12	39	31	5	36	12	30
	15	17	32	36	22	16	14	12
	17	38	16	33	23	30	29	16
Demand	20	13	11	27	9	5	40	

From the above table
The maximum {minimum of each row} and
The minimum maximum of each column is =16
Then using the steps 1- step 5, we have the following complete allocation table

Table 2

								Supply
	12	13		7	8		19	15
	7	18				6	10	7
	11	20		21	21			45
		12			5		12	30
	15	17				6	14	12
	17		16					16
Demand	20	13	11	17	9	5	40	

Now using step6, the optimal solution to the bottleneck problem is given below

Table-3

								Supply
					8(9)		19 (6)	15
						6 (5)	10(2)	7
	11(15)	20(3)		21(27)				45
							12(30)	30
		17(10)					14 (2)	12
	17(5)		16(11)					16
Demand	20	13	11	27	9	5	40	

From the table3, we conclude that minimum time transportation is 21.

3.1 Formulation of Fuzzy Bottleneck Transportation Problem (FBTP):

$$\text{Minimize } Z_1 = \sum_i^m \square \sum_j^n \square c_{ij} x_{ij}$$

$$\text{Minimize } Z_2 = [\text{maximize} T_{ij} / X_{ij} > 0]$$

$$\text{Subject to } \sum_j^n x_{ij} = a_i, \quad i=1,2,\ldots\ldots\ldots m \quad (1)$$

$$\sum_j^m x_{ij} = bj, \quad j=1,2,\ldots\ldots\ldots n \quad (1)$$

$$X_{ij} \geq 0 \; i=1,2,\ldots\ldots\ldots\ldots m; \; j=1,2,\ldots\ldots\ldots\ldots n;$$

Where a_i is the supply available at i^{th} sources b_j is the demand required at j^{th} destinations,

Xij= number of items while shipped from i^{th} source to j^{th} destination

Cij= cost of transportation a unit from i^{th} sours to j^{th} destinations,
T_{ij}= time of transportation goods from i^{th} source to j^{th} destination

3.2 Algorithms of blocking zero point approach:
Step1:construct the time transportation problem from the given BCTP.

Step 2: solve the time transportation problem by blocking method the optimal solution be To.

Step3: construct the cost transportation problem from the given BCTB

Step4: solve the cost transportation problem by the zero point method and also find the corresponding time transportation Tc

Step5: for each time Tα in [To, Tc], Compute $\beta = \frac{T_c - T_\infty}{T_c - T_0}$ which is the level of time satisfaction for the time

Step6: Construct the active cost transaction problem for each forTα is [To, Tc] and solve it by Zero point method.

Step7: For each time Tα an optional solving to the cost transportation problem, X_{13} obtained from the step 6 with level of time satisfaction 'β'

Then the vector (X ,Tα) is an efficient solution to BCTP.

3.3 Numerical Example

Consider the following 3x 4 BCTP.

Table 4

					Supply
	10 5	68 6	73 10	52 11	8
	66 6	95 7	30 12	21 14	19
	97 14	63 11	19 9	23 7	17
Demand	11	3	14	16	

The upper left corner is each cell give its time of transportation on the Corresponding route and its lower right corner is each cell gives the unit transportation cost / unit on that route.

Now, the transportation problem of BCTP is as follow.

Table 5

					Supply
	10	68	73	52	8
	66	95	30	21	19
	97	63	19	23	17
Demand	11	3	14	16	

Using its blocking method, we have that the optional solution of the time transportation problem of BTP is 66.

Now the cost transportation table of BCTP is given below

Table 6

					Supply
	5	6	10	11	8
	6	7	12	14	19
	14	11	9	7	17
Demand	11	3	14	16	

By Zero – point method. The solution is $X_{13} = 8; X_{21} = 11; X_{22} = 3; X_{23} = 5; X_{33} = 1; X_{14} = 16$ with the minimum transportation cost is 348 and the minimum time transportation is 95.

Now we have To =66, Tα = 95 and the time is Tα = {66, 68, 73, 95}
The active cost transportation problem of BCTP for Tα = 66 is given below

Table 7

					Supply
	5	-	-	11	8
	6	-	12	14	19
	-	11	9	7	17
Demand	11	3	14	16	

Using the zero point method the optional solution is X11 = 6 ; X14 = 2 ; X21 = 5 ; X23 = 14 ; X12 = 3 ; X14 = 14 With the total minimum transportation cost = 381.

Now the active cost transportation problem of BCTP for Tα = 73 is given below.

Table 8

					Supply
	5	6	10	11	8
	6	-	12	14	19
	-	11	9	7	17
Demand	11	3	14	16	

Using the zero point method the optional solution is $X_{12} = 3$; $X_{13} = 5$; $X_{21} = 11$; $X_{23} = 8$; $X_{33} = 1$; $X_{34} = 16$ With the total minimum transportation cost = 351.

The efficient solution to the BCTP is given below

Solution Table: 9

S.No	Efficient solution of BCTP	Objective Value Of BCTP	
1.	$X_{11} = 6; X_{14} = 2$ $X_{21} = 5; X_{23} = 14$ $X_{32} = 3; X_{34} = 14$ with Time =66	(381,66)	1
2.	$X_{11} = 5; X_{12} = 3$ $X_{21} = 6; X_{23} = 13$ $X_{33} = 1; X_{14} = 16$ With Time =68	(356,68)	27/29 = 0.93
3.	$X_{12} = 3; X_{13} = 5$ $X_{21} = 11; X_{23} = 8$ $X_{33} = 1; X_{14} = 16$ With Time = 73	(351,73)	22/29 = 0.76
4.	$X_{13} = 8; X_{21} = 11$ $X_{22} = 3; X_{23} = 5$ $X_{33} = 1; X_{34} = 16$ With Time 95	(348, 95)	

Conclusion:

In Its Paper We Have Proposed a Blacking Zero-Point Method to from the Optional Solution Of BiCriteria Bottleneck Transportation Problem With Fuzzy Parameters. We Obtain a requires Of Optional Solutions Based on the blocking Zero-Point Approach for a Two Relevant Numerical Example Were Presented to Enables, the Decision Maker to evaluate and correct the managerial decisions.

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